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PHYSICS-INFORMED NEURAL NETWORKS FOR INVERSE PROBLEMS IN TURBULENT AND MULTI-PHASE FLOWS: A CRITICAL ASSESSMENT OF PERFORMANCE, UNCERTAINTY, AND SCALABILITY

Olamide E. Danjuma ^{1, *} and Folake A. Adebayo ²

¹ Department of Mechanical Engineering, Faculty of Engineering, University of Lagos (UNILAG), Akoka, Lagos, Nigeria.

² Department of Mathematics, School of Physical Sciences, Federal University of Technology, Akure (FUTA), Nigeria.

* Corresponding Author

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ABSTRACT

Physics-Informed Neural Networks (PINNs) have emerged as a transformative paradigm for solving inverse problems in fluid mechanics, offering the ability to integrate sparse observational data with the governing physical laws encoded by the Navier-Stokes equations. This review critically evaluates the application of PINNs to inverse problems in turbulent and multi-phase flows—two of the most challenging regimes in computational fluid dynamics. We examine how embedding Navier-Stokes constraints into neural architectures enables the reconstruction of flow fields from limited measurements, a capability particularly valuable in oceanography, aerodynamics, and industrial process monitoring where dense experimental data are unavailable. The review systematically addresses three key sub-themes: uncertainty quantification in PINN predictions, the computational cost trade-offs between PINNs and traditional CFD solvers, and the development of hybrid PINN-CFD frameworks for industrial applications such as gas-liquid separators. Our analysis reveals that while PINNs demonstrate remarkable data efficiency and can outperform purely data-driven models in sparse-data scenarios, significant challenges persist in high-Reynolds-number turbulent flows, multi-phase systems with moving interfaces, and real-time industrial deployment. We identify critical research gaps, including the need for robust uncertainty estimation methods, scalable training algorithms for large-domain problems, and standardized benchmarking protocols. Finally, we offer perspectives on emerging trends, including self-adaptive PINNs, physics-informed operators, and the integration of digital twin frameworks for predictive industrial maintenance.

Keywords: Physics-Informed Neural Networks, Inverse problems, Turbulent flows, Multi-phase flows, Uncertainty quantification, Hybrid CFD-PINN

1 INTRODUCTION

The accurate prediction and reconstruction of fluid flows from limited observational data represents one of the most enduring challenges in computational fluid dynamics (CFD). In many practical scenarios—ranging from oceanographic monitoring and atmospheric modeling to aerospace design and industrial process control—direct measurements of flow fields are sparse, noisy, or confined to specific locations. Traditional CFD solvers, while capable of producing high-fidelity solutions, require complete specification of boundary conditions, initial conditions, and material properties—information that is often unavailable in real-world inverse problems.

The advent of Physics-Informed Neural Networks (PINNs), introduced by Raissi et al. (2019), has fundamentally altered this landscape. PINNs embed the governing partial differential equations (PDEs)—typically the Navier-Stokes equations for fluid flows—directly into the loss function of a neural network. This architecture enables the network to learn solutions that simultaneously satisfy the physical laws and fit available observational data. As noted by Sharma et al. (2023), physics-informed machine learning enables the integration of domain knowledge with machine learning algorithms, resulting in higher data efficiency and more stable predictions compared to purely data-driven approaches.

The significance of PINNs for inverse problems in turbulent and multi-phase flows cannot be overstated. In turbulent flows, the chaotic, multi-scale nature of the dynamics makes traditional data assimilation techniques computationally prohibitive. In multi-phase flows, the presence of moving interfaces, phase changes, and complex interfacial physics compounds the difficulty. PINNs offer a mesh-free, differentiable framework that can naturally incorporate sparse measurements—whether from particle image velocimetry (PIV), laser Doppler anemometry, or field sensors—to reconstruct complete flow fields and infer unknown parameters.

This review provides a comprehensive and critical assessment of PINN-based approaches for inverse problems in turbulent and multi-phase flows. We organize our analysis around three thematic pillars: (1) the reconstruction of turbulent flow fields from sparse observations, with emphasis on Reynolds-Averaged Navier-Stokes (RANS) formulations and high-Reynolds-number challenges; (2) uncertainty quantification in PINN predictions, addressing the critical need for confidence estimates in engineering decision-making; and (3) computational cost trade-offs and hybrid PINN-CFD frameworks for industrial applications, including gas-liquid separation systems. Throughout, we identify persistent limitations, unresolved questions, and promising directions for future research.

2 MAIN REVIEW

2.1 PINNs for Inverse Problems in Turbulent Flows

The application of PINNs to turbulent flows presents both exceptional opportunities and formidable challenges. Turbulence is characterized by a wide range of spatial and temporal scales, chaotic dynamics, and the closure problem inherent in RANS formulations. PINNs offer a compelling alternative to traditional data assimilation methods by leveraging the governing equations as soft constraints while flexibly incorporating sparse measurements.

2.1.1 RANS-Based Reconstruction from Sparse Data

The Reynolds-Averaged Navier-Stokes equations form the foundation of most engineering turbulence modeling. However, solving inverse problems with RANS using PINNs has proven non-trivial. Huang et al. (2023) investigated the application of PINNs to reconstruct the time-averaged turbulent flow field of a jet in crossflow from limited observations. Their study revealed that under limited observations, the application of PINNs was found to be difficult in solving the inverse problems of three-dimensional RANS equations. To address this, they imported the tensor-basis eddy viscosity model as prior knowledge into the PINN structure. After embedding this model, the highest absolute error and the relative L2 error of streamwise velocity were reduced by 11.1% and 31.4%, respectively. Remarkably, by designing a more effective training dataset containing only two planes and two pairs of lines—selected based on flow characteristics such as the shear layer and counter-rotating vortex pair—they achieved further error reductions of 30.0% and 6.4%, respectively.

This work highlights a critical insight: the success of PINNs for turbulent inverse problems depends not only on the network architecture and physics constraints but also on the strategic placement of observational data. The tensor-basis eddy viscosity model serves as a form of turbulence closure knowledge that regularizes the ill-posed inverse problem.

Xu et al. (2023) proposed a spatiotemporal parallel physics-informed neural network (STPINNs) framework specifically designed for inverse problems in fluid mechanics. The framework employs an overlapping domain decomposition strategy and incorporates RANS equations with eddy viscosity in the output layer of neural networks. The performance was evaluated on three turbulent cases: wake flow of a two-dimensional cylinder, homogeneous isotropic decaying turbulence, and the average wake flow of a three-dimensional cylinder. All three cases were successfully reconstructed with sparse observations, and quantitative results demonstrated that STPINNs can accurately and efficiently solve turbulent flows with comparatively high Reynolds numbers.

The STPINNs framework addresses a fundamental limitation of standard PINNs: the difficulty in handling large spatiotemporal domains and high Reynolds numbers, which leads to hyper-parameter tuning difficulties and excessively long training times. By parallelizing the computation across multiple processing units, STPINNs enable the reconstruction of turbulent flows that would be impractical with conventional PINN architectures.

Table 1: Summary of Key PINN Studies for Inverse Problems in Turbulent and Multi-Phase Flows

Study	Flow Regime	Key Contribution	PINN Variant	Performance Highlight	Limitation Identified
Raissi et al. (2019)	General PDE systems	Foundational PINN framework for forward/inverse problems	Standard PINN	Demonstrated feasibility for Navier-Stokes inverse problems	Limited to moderate Reynolds numbers
Huang et al. (2023)	Jet in crossflow (turbulent, 3D RANS)	Tensor-basis eddy viscosity embedding; strategic sensor placement	Standard PINN + t-EV model	31.4% reduction in L2 error with t-EV model; further 30% reduction with optimized sensor placement	Difficulty with 3D RANS inverse problems under limited observations
Xu et al. (2023)	Cylinder wakes, decaying turbulence (high Re)	Spatiotemporal parallel PINN with domain decomposition	STPINNs (overlapping domain decomposition)	Successful reconstruction of all three turbulent cases with sparse observations	Requires multi-CPU deployment; hyper-parameter tuning remains challenging
Xiao et al. (2023)	Rayleigh-Taylor turbulent mixing	Improved PINN framework for RANS modeling of turbulent instability	RANS-constrained PINN	Effective reconstruction of turbulent mixing induced by RT instability	Specific to RT instability; generalizability unclear
Fraces and Tchelepi (2023)	Immiscible two-phase flow in porous media	Parameterized PINN for uncertainty quantification	P-PINN	Matches ensemble realizations and stochastic moments	Computational cost scales with parameter space dimensionality
Yan et al. (2023)	Multiphase flow in fractured porous media	Physics-informed convolutional neural network	PICNN	Accurate capture of fracture effects on multiphase flow	Limited to porous media applications
Soibam et al. (2024)	Mixed convection around square cylinder	Ensemble PINN for uncertainty quantification	Ensemble PINN	Identifies regions of reliable prediction and potential inaccuracies	Increased computational cost due to ensemble training
Carvalho et al. (2023)	Centrifugal pumps under multi-phase flow	Learning characteristic parameters and dynamics	Standard PINN	Demonstrates industrial equipment monitoring potential	Early-stage development; limited validation
Lino et al. (2023)	Various fluid dynamics problems	Comprehensive review of deep learning methods for fluid simulation	Various	Systematic comparison of emerging methods	Conceptual review; no new experimental results

2.1.2 Turbulence Model Augmentation

An emerging paradigm involves augmenting PINNs with turbulence models to improve reconstruction accuracy. Recent work has demonstrated that turbulence-model-augmented PINNs can achieve substantial error reductions—up to 73% reduction in mean velocity reconstruction error with coarse measurements. These approaches bridge the gap between purely data-driven and purely physics-based methods by using RANS equations constrained by accurate sparse pointwise mean velocity measurements for data assimilation.

For the Rayleigh-Taylor turbulent mixing problem, Xiao et al. (2023) proposed an improved PINNs framework for solving RANS equations. This work exemplifies the growing trend of tailoring PINN architectures to specific turbulence phenomena, recognizing that the standard PINN formulation may be insufficient for capturing the complex physics of turbulent mixing.

2.2 PINNs for Multi-Phase Flows

Multi-phase flows introduce additional complexity due to the presence of moving interfaces, phase transitions, and coupled transport phenomena. The application of PINNs to multi-phase systems is an emerging area with significant industrial relevance.

2.2.1 Two-Phase Flow in Porous Media

The Buckley-Leverett problem, describing immiscible two-phase flow displacement in porous media, has been a canonical test case for PINN-based multi-phase flow modeling. Fraces and Tchelepi (2023) presented a Parameterized Physics-Informed Neural Network (P-PINN) approach for uncertainty quantification in reservoir engineering problems. By treating reservoir properties such as porosity and permeability as random variables, they demonstrated that PINNs can produce solutions that match closely both ensemble realizations and stochastic moments. The additional dimensions resulting from stochastic treatment of the PDEs tend to produce smoother solutions on quantities of interest, which improves PINN performance.

Yan et al. (2023) developed a Physics-Informed Convolutional Neural Network (PICNN) for solving multiphase flow in fractured porous media. Their work demonstrated the accuracy of the proposed approach in capturing the impact of fractures on multiphase flow. The convolutional architecture leverages spatial locality to efficiently represent the complex geometry of fractured media.

2.2.2 Dynamic Two-Phase Interface Problems

The simulation of dynamic two-phase interfaces presents unique challenges for PINNs due to the discontinuity in material properties and the moving boundary. Recent work has explored meshfree methods using deep neural networks for solving dynamic two-phase interface problems. These approaches show promise for applications where traditional interface-tracking methods are computationally expensive or difficult to implement.

Carvalho et al. (2023) investigated the use of PINNs for learning characteristic parameters and dynamics of centrifugal pumps under multi-phase flow. This application demonstrates the potential of PINNs for industrial equipment monitoring, where the ability to infer system parameters from limited measurements is of practical value.

2.3 Uncertainty Quantification in PINN Predictions

For PINNs to be adopted in engineering practice, reliable uncertainty estimates are essential. Unlike traditional numerical solvers, neural networks do not inherently provide confidence intervals for their predictions.

2.3.1 Ensemble PINNs

Soibam et al. (2024) implemented an ensemble physics-informed neural network approach to tackle ill-posed problems with unknown thermal boundaries and limited sensor data. The ensemble approach mitigates the risks of overfitting and underfitting while providing a measure of model confidence. As a result, the ensemble model can identify regions of reliable prediction and potential inaccuracies. This capability is critical for applications where incorrect predictions could have safety or economic consequences.

The ensemble approach builds upon the observation that multiple PINNs trained with different initializations or data subsets can provide a distribution of predictions, from which uncertainty intervals can be derived. While computationally more expensive than single-network training, ensemble methods offer a practical path toward uncertainty-aware PINN predictions.

2.3.2 Parameterized Approaches for Stochastic Problems

The P-PINN approach of Fraces and Tchelepi (2023) represents an alternative strategy for uncertainty quantification. By parameterizing the uncertainty space and treating uncertain parameters as additional input dimensions, the network learns to map the entire parameter space to solution fields. This approach enables the efficient computation of stochastic moments and can compete in performance with traditional stochastic sampling methods.

2.4 Computational Cost Trade-offs

A critical consideration in evaluating PINNs for industrial applications is the computational cost relative to traditional CFD solvers. The trade-offs are not straightforward and depend on the specific problem, the available data, and the computational resources.

2.4.1 Memory and Time Comparisons

Recent studies have quantified the computational trade-offs between PINNs and CFD. In general, PINNs are much more efficient than CFD in terms of memory resource usage, requiring 5–10 times less memory. However, this memory advantage comes at the cost of longer computational time, with PINNs requiring approximately 3 times more computational time than CFD.

This trade-off reflects the fundamentally different computational paradigms: CFD solvers use efficient, specialized numerical methods for specific equation systems, while PINNs perform gradient-based optimization over a large parameter space. The memory efficiency of PINNs is particularly valuable for problems where memory constraints are limiting, such as large-domain simulations or computations on graphics processing units with limited memory capacity.

2.4.2 Hybrid Approaches for Cost Reduction

The finite difference-based PINN (FD-PINN) approach offers potential computational savings by decoupling the predictive accuracy of derivatives from the network's approximating accuracy. FD-PINNs can save computational costs when large-size networks or meshes are used. However, as noted by Jiang et al. (2023), FD-PINNs increase computational cost when handling random point distributions, especially with higher-order discretization schemes.

Alternative approaches, such as the radial basis function-differential quadrature-based PINN, have been proposed for steady incompressible flows. While expedient in some cases, these methods can be time-consuming with higher-order derivatives and may lead to nonphysical solutions with sparse samples.

Table 2: Computational Cost Trade-offs: PINNs vs. Traditional CFD

Metric	PINNs	Traditional CFD	Advantage	Key References
Memory Usage	5–10× less	Higher	PINNs	Sharma et al. (2023)
Computational Time	~3× more	Lower	CFD	Sharma et al. (2023)
Mesh Generation	Not required (mesh-free)	Required; time-consuming for complex geometries	PINNs	Lino et al. (2023)
Parallelization	Emerging (STPINNs)	Well-established (domain decomposition)	CFD	Xu et al. (2023)
Data Efficiency	High (can incorporate sparse data)	Low (requires full boundary/initial conditions)	PINNs	Raissi et al. (2019)
Derivative Computation	Automatic differentiation; expensive for higher-order derivatives	Numerical differentiation; efficient for structured grids	CFD (for structured grids)	Jiang et al. (2023)
GPU Acceleration	Highly compatible	Variable	PINNs	Multiple sources
Surrogate Modeling	Natural (trained network is a surrogate)	Requires additional model reduction	PINNs	Lino et al. (2023)
Handling Complex Geometries	Flexible; mesh-free	Requires mesh generation; can be challenging	PINNs	Yan et al. (2023)
Uncertainty Quantification	Can be integrated (ensemble, parameterized)	Requires separate methods	PINNs	Soibam et al. (2024); Fraces and Tchelepi (2023)

2.5 Hybrid PINN-CFD Solvers for Industrial Applications

The integration of PINNs with traditional CFD solvers represents a pragmatic path toward industrial adoption. Rather than replacing CFD entirely, PINNs can augment CFD simulations in specific ways.

2.5.1 Data Assimilation and Parameter Identification

PINNs have been applied to enhance PIV measurements, demonstrating their utility in improving experimental data quality through physics-informed regularization. The ability to assimilate sparse experimental data into physically consistent flow fields is valuable for applications ranging from wind tunnel testing to field measurements.

For industrial systems such as pipe conveying fluid systems, PINN-based digital parameter identification has been explored for monitoring, maintenance planning, and operation optimization. These applications leverage the ability of PINNs to infer unknown parameters from limited measurements, reducing the need for extensive instrumentation.

2.5.2 Turbomachinery and Compressor Applications

Recent work has applied PINNs to predict flow fields in compressor cascades. This research provides compelling evidence that PINNs offer turbomachinery designers an additional and promising option alongside traditional CFD methods. The mesh-free nature of PINNs is particularly advantageous for complex geometries where mesh generation is time-consuming.

2.5.3 Gas-Liquid Separators

While specific applications to gas-liquid separators remain limited in the peer-reviewed literature, the principles demonstrated in other multi-phase systems are directly transferable. The ability of PINNs to incorporate sparse measurements—such as pressure drops, liquid holdup, and phase fraction measurements—to reconstruct complete flow fields offers significant potential for optimizing separator design and operation. Hybrid approaches that use CFD to generate training data for PINNs, which then serve as fast surrogate models for design optimization, represent a promising direction.

3 FUTURE PERSPECTIVES

Several emerging trends and unresolved questions promise to shape the future of PINN-based inverse problems in turbulent and multi-phase flows.

Self-adaptive and attention-based PINNs represent a significant frontier. Current PINN formulations use fixed weights for the different loss components (PDE residual, data loss, boundary conditions), leading to challenging hyper-parameter tuning. Self-adaptive PINNs that automatically balance these loss components during training could dramatically improve performance and reduce the expertise required for PINN deployment. Attention mechanisms, which allow the network to focus on regions of high error or complex physics, offer another promising direction.

Physics-informed operator learning—exemplified by Fourier Neural Operators and Deep Operator Networks—extends the PINN paradigm to learning solution operators rather than individual solutions. This approach enables the rapid evaluation of solutions for new parameter values without retraining, offering transformative potential for parametric studies, optimization, and uncertainty quantification. For turbulent flows, operator learning could enable real-time predictions for varying Reynolds numbers or boundary conditions.

Digital twins and predictive maintenance represent high-value industrial applications. The integration of PINNs with sensor data from operating equipment—including gas-liquid separators, pumps, and compressors—could enable real-time monitoring, fault detection, and predictive maintenance. The ability of PINNs to infer unmeasured quantities from sparse sensor data is particularly valuable for industrial systems where comprehensive instrumentation is impractical.

Standardized benchmarking and validation protocols are urgently needed. The field currently lacks consensus on how to evaluate and compare PINN performance for inverse problems in fluid mechanics. Standardized test cases, error metrics, and reporting guidelines would accelerate progress and enable meaningful comparisons across studies.

Hybrid quantum-physics-informed neural networks have emerged as a speculative but intriguing direction. Recent work has demonstrated hybrid quantum PINNs that simulate laminar fluid flows, achieving higher accuracy compared to purely classical neural networks. While still in early stages, quantum-enhanced PINNs could offer computational advantages for large-scale problems.

In situ and real-time training remains a significant challenge. Most PINN applications require offline training, which can take hours to days. For real-time applications—such as active flow control or emergency response—the training time must be dramatically reduced. Advances in neural network architectures, training algorithms, and hardware acceleration are needed to enable real-time PINN deployment.

Multi-fidelity and transfer learning approaches offer opportunities to leverage existing CFD databases to accelerate PINN training. By pretraining on synthetic data from high-fidelity simulations and fine-tuning with sparse experimental measurements, PINNs could achieve high accuracy with minimal experimental data.

Improved handling of discontinuities and shocks remains a critical challenge for PINNs in compressible and multi-phase flows. The standard PINN formulation struggles with sharp gradients and discontinuities. Physics-informed shock capturing methods and specialized architectures that incorporate discontinuity-detection mechanisms are promising research directions.

4 CONCLUSION

Physics-Informed Neural Networks have emerged as a powerful and versatile framework for solving inverse problems in turbulent and multi-phase flows. This review has demonstrated that PINNs offer compelling advantages over purely data-driven approaches in scenarios with sparse measurements, leveraging the governing Navier-Stokes equations as strong inductive biases that regularize the ill-posed inverse problem.

The application of PINNs to turbulent flows has shown particular promise, with studies demonstrating the reconstruction of turbulent wake flows, decaying turbulence, and jet-in-crossflow configurations from limited observations. The integration of turbulence models, such as tensor-basis eddy viscosity formulations, has been shown to substantially improve reconstruction accuracy. However, significant challenges persist, particularly for three-dimensional RANS inverse problems and high-Reynolds-number flows where the standard PINN formulation struggles with training convergence and solution accuracy.

For multi-phase flows, PINNs have demonstrated capabilities in two-phase flow in porous media, fractured media, and dynamic interface problems. The ability to incorporate uncertainty quantification through ensemble methods and parameterized approaches adds to the practical utility of PINNs for engineering applications.

The computational cost trade-offs between PINNs and traditional CFD solvers are nuanced: PINNs require 5–10 times less memory but approximately 3 times more computational time. This trade-off favors PINNs in memory-constrained environments and for problems where the cost of mesh generation is high. Hybrid PINN-CFD approaches, where PINNs augment rather than replace traditional solvers, represent a pragmatic path toward industrial adoption.

Looking forward, the integration of PINNs with digital twin frameworks, the development of self-adaptive and attention-based architectures, and the emergence of physics-informed operator learning promise to expand the capabilities and applicability of PINNs for turbulent and multi-phase flows. The realization of this potential will require continued advances in algorithmic development, standardized benchmarking, and validation against experimental data. As the field matures, PINNs are poised to become an indispensable tool for inverse problems in fluid mechanics, enabling the extraction of physical insight from sparse data and accelerating the design and optimization of engineering systems.

STATEMENTS ON COMPLIANCE WITH ETHICAL STANDARDS

Disclosure of conflict of interest

The authors declare no conflicts of interest regarding this manuscript.

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